

MATH 141: Some Practice Final Problems

Here are problems that cover the last two weeks of our class.

The final is cumulative; you should look at Practice Midterm 1+2 and Midterm 1+2 as well.

1. Compute this integral:

$$\int_0^1 \left(\sum_{i=1}^6 (i+1)x^i \right) dx$$

2. Evaluate the following expressions. If applicable, you are allowed to use Fundamental Theorem of Calculus.

(a) $\sum_{i=1}^5 \frac{f(i)}{i}$ given that $f(x) = x^2$

(b) $\frac{d}{d\theta} \int_{-\pi}^{\theta} \sin x \, dx$

(c) $\int_1^5 \frac{x^2 - 1}{x - 1} \, dx$

(d) $\lim_{x \rightarrow \infty} \frac{x+1}{\sqrt{x^4-2}}$

(e) $\int (\sec(x) \tan(x) - \csc(x) \cot(x)) \, dx$

(f) $\lim_{x \rightarrow \infty} (x^5 - x^3)$

(g) $\int_{-1}^1 \frac{3x^2 + 4x + 4}{x} dx$

(h) $\int_0^6 |x - 3| dx$

3. Suppose $f(x) = x^2$. Approximate the area underneath the curve on the interval $[1, 2]$ using four rectangles and right endpoints.

Only set up the sum; do not compute it.

4. Consider the functions

$$g(x) = \int_0^x t^2 dt \qquad h(x) = \int_0^x \sin(t^3) dt$$

(a) What is the geometric meaning of the number $g(5)$?

(b) What is the geometric meaning of the number $h(3)$?

(c) Evaluate the following expression:

$$\frac{d}{dx}[2g(x) + 3h(x)]$$

5. A particle is traveling along a horizontal line. The instantaneous velocity is

$$v(t) = t^2 - 2t - 3$$

- (a) Draw a visualization of the particle with both negative and positive velocity.

- (b) Determine the total displacement on the time interval $[2, 4]$. Did the particle move to the left or the right of the starting point?

- (c) Now determine the total distance traveled on $[2, 4]$.

6. Evaluate and fully simplify the following:

(a) $\int \sin^2 \theta \cos \theta \, d\theta$

(b) $\int_0^{\sqrt{\pi}} x \cos(x^2) \, dx$

(c) $\int x^2 \sqrt{x+1} \, dx$

(d) $\int_0^1 \cos\left(\frac{\pi t}{2}\right) \, dt$